THE ASSOCIATION OF RICHARDSON'S CRITERION WITH HIGH LEVEL TURBULENCE

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ABSTRACT

Investigations have shown various meteorological parameters which could be associated with high level turbulence. Two of these parameters are strong vertical wind shear and low stability. These and the Richardson Number are analyzed with respect to geographical location, proximity to the level of maximum wind, proximity to the tropopause, and seasons. The purpose of this study is to find out under what conditions the correlation between high level turbulence and low Richardson Number was good and under what conditions the correlation was poor. It is found for the most part that using the Richardson Number as a predictor of high level turbulence appears to be more successful in the eastern half of the country.

1. INTRODUCTION

Investigations have shown various parameters which could be associated with High Level Turbulence (HLT), and one of these parameters is the Richardson Number (Ri). Ri is defined [6],[9] as

$$Ri=\frac{-C}{V}=\frac{g\Delta\theta}{\Delta z}\Big/\bar{\theta}\left(\frac{\Delta u}{\Delta z}\right)^2,$$

where C is a measure of the rate of production of eddy energy by vertical temperature difference, V is a measure of the rate of transfer of eddy energy by vertical wind shear, g is the acceleration of gravity, $\bar{\theta}$ is the mean potential temperature of the layer, $\Delta\theta/\Delta z$ is the thermal stability (potential temperature lapse rate, ° C. per 1,000 ft.) and $\Delta u/\Delta z$ is the vertical wind shear (ft. per 1,000 ft.).

The quantity $-C$ is a positive number which measures the rate of destruction of turbulent energy, because turbulent motion is doing work against the stable stratification. The quantity V is the main energy transfer term. $-C/V$ measures the ratio of the rate work is done against gravitational (thermal) stability to the rate energy is transferred by eddy stresses from mean to turbulent motion. The smaller the Ri, the more likely is the energy in local turbulent eddies to increase.

Considerable work has been attempted in the past 10 years to correlate the occurrence of HLT with Ri. These results have been very erratic. Anderson [1], Berenger and Heissat [2], Pinus [10], and others found good correlation between low Ri and HLT occurrences. While on the other hand Brundidge [3], Lake [7], Mook [8], Panofsky [9], and others found poor correlation. This lack of agreement has been partly due to the lack of a large enough sample of HLT occurrences to use, and not having the HLT occurrences within a reasonable proximity in time and space to a sounding release.

This problem of lack of data was eased when the Clear Air Turbulence Project of the U.S. Weather Bureau collected what the author believes at the time of this writing is the largest number of HLT reports ever available for a study of this nature. From August 1, 1960, through July 31, 1961, approximately 10,000 pilot reports of HLT were received from the 27 Weather Bureau Airway Forecast Centers, commercial airlines, and military services. These were reports of turbulence at or above 15,000 ft. and with a reported intensity of moderate or greater. The Clear Air Turbulence Project screened the reports to eliminate those which could possibly be the results of convective activity. This screening process reduced the usable number of reports to 6,488.

Because of the large number of HLT reports available, only those HLT occurrences within 100 mi. of a sounding station and within a time limit of ± 1 hr. of sounding time are used in this study. This second screening process is considered necessary in order to gain as true a picture of the atmosphere as possible at the place and time of a HLT occurrence. If these limitations are not imposed, interpolation and extrapolation are required and this in turn subtracts from the value of the final results. This final screening process reduced the usable number of reports to 162.

For each of the 162 HLT occurrences the nearest sounding was analyzed for vertical wind shear, stability, and mean potential temperature for the layer in which the HLT occurred. The layer was considered 2,000 ft.

thick, 1,000 ft. below and above the level of the HLT occurrence.

The vertical wind shear, stability, and *Ri* for each of the reports were then studied with respect to the following parameters:

1. *Geographical location.*—As suggested by Colson [5], an attempt was made to see what influence, if any, the large mountain ranges of the western United States might have on the probable HLT causes of strong vertical wind shear and low stability, and subsequently the resultant *Ri*. Therefore, the 102° W. longitude line was chosen as the dividing line between the large mountain ranges in the west, and the plains and the relatively smaller mountains and hills in the east. Any cases falling on the line were considered eastern cases.

2. *Tropopause.*—The data were tabulated by 2,000-ft. layers above and below the tropopause to see if there are any preferred locations with respect to the tropopause for the occurrence of HLT.

3. *Level of Maximum Wind.*—The data were tabulated by 2,000-ft layers above and below the level of maximum wind for preferred locations of HLT occurrence.

4. *Season.*—The data were analyzed to see if there were any variations by season in the number of HLT cases, or in the magnitude of the vertical wind shear, stability, and resultant *Ri*. The seasons are defined as follows: winter—January through March; spring—April through June; summer—July through September; and fall—October through December.

One parameter which was reluctantly excluded from this study was the type of aircraft encountering the HLT, since for security reasons or from carelessness the type of aircraft was often omitted on the reports.

2. INVESTIGATION OF *Ri*, VERTICAL WIND SHEAR, AND STABILITY

GEOGRAPHICAL LOCATION

A tabulation of the 162 cases of HLT was made with respect to geographical location and four categories of *Ri*, and is shown in table 1. Of the 162 cases of HLT, 93 fell east of the 102° W. line and 69 cases fell west of the line. This larger number of cases in the east is probably due to the fact that there is a considerably larger area east of the line, and also there are more sounding stations east of the line.

Table 1.—*Tabulation of* Ri *by location*

	East		West		All cases	
	Number of cases	Percent	Number of cases	Percent	Number of cases	Percent
$Ri \leq 1$	22	23.7	10	14.5	32	19.7
$1 < Ri \leq 5$	38	40.8	23	33.3	61	37.7
$5 < Ri \leq 10$	11	11.8	12	17.4	23	14.2
$Ri > 10$	22	23.7	24	34.8	46	28.4
Total	93	---------	69	---------	162	---------

Table 2.—*Mean values of vertical wind shear (kt./1,000 ft.), lapse rate (° C./1,000 ft.), and* Ri *by location. For the computation of the mean* Ri *values, 320° C. was used as the mean potential temperature of the layer*

Mean values of—	East	West	All cases
$\|\Delta u/\Delta z\|$	5.3	4.0	4.8
$\Delta\theta/\Delta z$	2.3	2.7	2.4
Ri	2.9	5.9	3.7

East of 102° W., 65 percent of the occurrences fell into the two lower *Ri* groups ($Ri \leq 1$ and $1 < Ri \leq 5$), and only 35 percent fell in the two higher *Ri* groups ($5 < Ri \leq 10$ and $Ri > 10$). West of 102° W. there is a more equal division with 48 percent in the lower two *Ri* groups and 52 percent in the higher two *Ri* groups. It is interesting to note that 35 percent of the western cases occurred in the highest *Ri* group ($Ri > 10$). This suggests that the mechanism associated with *Ri* is not as important over the higher mountains of the west. This is in agreement with work in mountain wave development. This can also be seen in the mean values obtained for the vertical wind shear and stability shown in table 2. The higher mean vertical wind shear and slightly lower mean stability in the east results in a mean *Ri* of 2.9, while the lower mean vertical wind shear and slightly higher mean stability in the west results in a mean *Ri* of 5.9. Thus the correlation between HLT and *Ri* is much better in the eastern half of the country.

LEVEL OF MAXIMUM WIND

Distance above and below the level of maximum wind.—An analysis of the 162 cases with respect to the four *Ri* groups and to the 2,000-ft. layers above and below the level of maximum wind is presented in table 3. There is a relative'y deep layer extending from 10,000 ft. below to 4,000 ft. above the level of maximum wind in which there is a fairly uniformly large number of cases for each 2,000-ft. layer. Above and below this 14,000-ft. layer

Table 3.—*Tabulation of cases with respect to* Ri *and distance above and below level of maximum wind*

	(In 1,000's of ft.)	$Ri \leq 1$	$1 < Ri \leq 5$	$5 < Ri \leq 10$	$Ri > 10$	Totals
Above level of maximum wind	>10		3		2	5
	8 to 10				1	1
	6 to 8		2		2	4
	4 to 6		3	1	4	8
	2 to 4	5	4	3	2	14
	0 to 2#		2	2	5	9
Below level of maximum wind	0 to 2*	5	10	5	8	28
	2 to 4	4	4	2	4	14
	4 to 6	4	12	1	1	18
	6 to 8	6	4		3	13
	8 to 10	3	9	1	1	14
	10 to 12		2	1	1	4
	12 to 14			2	2	4
	14 to 16	1	1	1	1	4
	16 to 18			2	2	4
	>18	4	5	2	7	18
	Total	32	61	23	46	162

#Excluding cases occurring at level of maximum wind.
*Including cases occurring at level of maximum wind.

the drop-off of cases is fairly large. The maximum number of occurrences, 28, is in the 2,000-ft. layer just below the level of maximum wind.

Excluding two occurrences which were at 22,000 ft. above the level of maximum winds and a single occurrence at 35,000 ft. below the level of maximum wind, all of the cases fell within a 44,000-ft. layer extending 28,000 ft. below to 16,000 ft. above the level of maximum winds. It is desirable to reduce this layer to a smaller thickness and yet include as large a number of the HLT cases as possible within the layer. A 14,000-ft. layer, extending 4,000 ft. above and 10,000 ft. below the level of maximum wind was arbitrarily chosen. This is marked off by the bold rules in table 3. Of the 162 HLT cases 110 or about 68 percent fell within this 14,000-ft. layer. Also within this layer, 72 cases were in the lower two *Ri* groups compared to 38 cases in the higher two *Ri* groups. Above and below this layer, there were 21 cases in the lower two *Ri* groups and 31 in the higher two *Ri* groups. Thus, the correlation of HLT to low *Ri* is best in the layer from 4,000 ft. above to 10,000 ft. below the level of maximum wind.

Mean values of the vertical wind shear and stability are shown in table 4. From these mean values the mean *Ri* values were computed. There was not too much difference in the mean vertical wind shear values of 4.7 kt. per 1,000 ft. below, and 5.1 kt. per 1,000 ft. above the level of maximum wind. But as might be expected, the difference in the mean stability values was obvious with the more stable lapse rate of 3.8° C. per 1,000 ft. above the level of maximum wind, and the least stable lapse rate of 2.0° C. per 1,000 ft. below the level of maximum wind. This increased stability above the level of maximum wind is primarily caused by stratospheric air which is more often encountered above the level of maximum wind than below it. Even though there was a slightly larger mean vertical wind shear above than below the level of maximum wind, this was more than offset by the less stable air below the level of maximum wind. The resultant mean *Ri* values for above and below the level of maximum wind are 5.1 and 3.2 respectively.

Distance above or below the level of maximum wind in the troposphere.—An analysis of the 126 cases which occurred below the tropopause was made with respect to the four *Ri* groups and distance above or below the level of maximum wind as shown in table 5. Considering all 126 of the HLT cases, 81 or about 64 percent occur in the lower two *Ri* groups, and the remaining 45, or about 36 percent occur in the higher two *Ri* groups.

There is a 10,000-ft. layer (bold rules in table 5) extending 10,000 ft. below the level of maximum wind in which there is a large number of cases ranging from 11 to 20 for each 2,000-ft. layer. Above and below this 10,000-ft. layer the drop-off of cases is fairly large. The maximum number of occurrences, 20, is in the 2,000-ft. layer just below the level of maximum wind.

Of the 126 HLT cases, 75, or about 60 percent, occurred within this 10,000-ft. layer below the level of maximum wind. Within this layer, *Ri* is quite an effective criterion. A total of 59, or about 79 percent of the 75 HLT cases within the layer occurred in the lower two *Ri* groups, with the remaining 16, or 21 percent occurring in the higher two *Ri* groups. These results indicate a good correlation of HLT and low *Ri* within this 10,000-ft. layer.

The mean values of vertical wind shear and stability, and the computed mean values of *Ri* are shown in table 6. Of the 126 HLT cases 108 occurred below and only 18 occurred above the level of maximum wind. The mean *Ri* of 2.3 below is considerably less than the mean *Ri* of 5.4 above the level of maximum wind. This lower mean *Ri* below the level of maximum wind is due to both the larger mean vertical wind shear of 4.8 kt. per 1,000 ft.

TABLE 5.—*Tabulation of cases below the tropopause with respect to* Ri *and distance above or below the level of maximum wind*

	(In 1,000's of ft.)	$Ri \leq 1$	$1 < Ri \leq 5$	$5 < Ri \leq 10$	$Ri > 10$	Totals
Above level of maximum wind	>10		1		1	2
	8 to 10				1	1
	6 to 8		2			2
	4 to 6				3	3
	2 to 4	3	1			4
	0 to 2#		2	1	3	6
Below level of maximum wind	0 to 2*	5	9	3	3	20
	2 to 4	3	4	1	3	11
	4 to 6	4	12	1	1	18
	6 to 8	6	4		2	12
	8 to 10	3	9	1	1	14
	10 to 12		2	1	1	4
	12 to 14			1	2	3
	14 to 16	1	1	1	1	4
	16 to 18			2	2	4
	18 to 20	1	3		2	6
	20 to 22				3	3
	22 to 24		1	2		3
	24 to 26	3	1			4
	>26				2	2
	Total	29	52	14	31	126

#Excluding cases occurring at level of maximum wind.
*Including cases occurring at level of maximum wind.

TABLE 4.—*Means for the vertical wind shear values (kt./1,000 ft.), lapse rate values (° C./1,000 ft.), and* Ri *with respect to the level of maximum wind. For the computation of the mean* Ri *values, 320° C. was used as the mean potential temperature of the layer*

	Means of—			Number of cases	Percent of all cases
	$\left\lvert\frac{\Delta u}{\Delta z}\right\rvert$	$\frac{\Delta\theta}{\Delta z}$	Ri		
Below level of maximum wind	4.7	2.0	3.2	121	74.7
Above level of maximum wind	5.1	3.8	5.1	41	25.3

TABLE 6.—*Mean values below tropopause for the vertical wind shear (kt./1,000 ft.), lapse rate (° C./1,000 ft.), and* Ri *with respect to level of maximum wind. For the computation of the mean* Ri *values, 320° C. was used as the mean potential temperature of the layer*

	Means of—			Number of cases	Percent of all cases
	$\left\lvert\frac{\Delta u}{\Delta z}\right\rvert$	$\frac{\Delta\theta}{\Delta z}$	Ri		
Below level of maximum wind	4.8	1.5	2.3	108	66.7
Above level of maximum wind	3.5	1.9	5.4	18	11.1

TABLE 7.—*Tabulation of cases above the tropopause with respect to* Ri *and distance above or below the level of maximum wind*

	(In 1,000's of ft.)	$Ri \leq 1$	$1 < Ri \leq 5$	$5 < Ri \leq 10$	$Ri > 10$	Totals
Above level of maximum wind	>8		2		1	3
	6 to 8				2	2
	4 to 6		3	1	1	5
	2 to 4	2	3	3	2	10
	0 to 2#			1	2	3
Below level of maximum wind	0 to 2*		1	2	5	8
	2 to 4	1		1	1	3
	4 to 6					
	6 to 8				1	1
	>8			1		1
	Total	3	9	9	15	36

#Excluding cases occurring at level of maximum wind.
*Including cases occuring at level of maximum wind.

TABLE 9.—*Tabulation of cases with respect to* Ri *and distance above and below tropopause*

	(In 1,000's of ft.)	$Ri \leq 1$	$1 < Ri \leq 5$	$5 < Ri \leq 10$	$Ri > 10$	Totals
Above tropopause	>4			2	2	4
	2 to 4	2	5	2	6	15
	0 to 2#	1	4	5	7	17
Below tropopause	0 to 2*	3	9		3	15
	2 to 4	3	5	4	3	15
	4 to 6	2	6	1	7	16
	6 to 8	6	3	2	2	13
	8 to 10	2	9	2	1	14
	10 to 12	1	2	1	4	8
	12 to 14	2	1		4	7
	14 to 16	1	2	1		4
	16 to 18	2	4	1	2	9
	18 to 20	2	2		1	5
	20 to 22		3		3	6
	22 to 24	3	2	1	1	7
	>24	2	4	1		7
	Total	32	61	23	46	162

#Excluding cases occurring at level of the tropopause.
*Including cases occurring at level of the tropopause.

and the lower mean stability of 1.5° C. per 1,000 ft. as compared to a smaller mean vertical wind shear of 3.5 kt. per 1,000 ft. and a slightly higher mean stability of 1.9° C. per 1,000 ft. above the level of maximum wind.

Distance above or below the level of maximum wind in the stratosphere.—An analysis of the 36 cases which occurred above the tropopause was made with respect to the four *Ri* groups, and distance above or below the level of maximum wind. The results of this analysis are shown in table 7.

Of the 35 cases, 29, or about 81 percent occurred between 4,000 ft. below and 6,000 ft. above the level of maximum wind, (bold rules in table 7) with the greatest number, 10, occurring 2,000 to 4,000 ft. above the level of maximum wind. Only 10, or about 34 percent of the 29 cases occurred in the lower two *Ri* groups, and the remaining 19 cases, or about 66 percent occurred in the higher two *Ri* groups.

Considering all of the 36 cases, only 3, or 8 percent had $Ri \leq 1$, and 12, or 33 percent had $Ri \leq 5$. This of course is a poor correlation between low *Ri* and HLT occurrence. This is largely due to greatly increased stability above the tropopause.

The mean values of vertical wind shear and stability and the computed mean values of *Ri* are shown in table 8. Of the 36 cases, 23 occurred above and 13 occurred below the level of maximum wind. The mean *Ri* of 4.6 above is considerably less than the mean *Ri* of 17.9 below the level of maximum wind. This is due primarily to a larger mean vertical wind shear of 6.3 kt. per 1,000 ft. above as compared to 3.6 kt. per 1,000 ft. below the level of maximum wind. Though the mean stability above and below the level of maximum wind was relatively high, 5.2° and 6.6° C. per 1,000 ft. respectively, the slightly lower stability above the level of maximum wind also helped to bring about the lower mean *Ri* above the level of maximum wind.

TABLE 8.—*Mean values above tropopause for the vertical wind shear (kt./1,000 ft.), lapse rate (° C./1,000 ft.), and* Ri *with respect to level of maximum wind. For the computation of the mean* Ri *values, 320° C. was used as the mean potential temperature of the layer*

	Means of—			Number of cases	Percent of all cases
	$\left\lvert\frac{\Delta u}{\Delta z}\right\rvert$	$\frac{\Delta\theta}{\Delta z}$	Ri		
Below level of maximum wind	3.6	6.6	17.9	13	8.0
Above level of maximum wind	6.3	5.2	4.6	23	14.2

TROPOPAUSE

Distance above and below the tropopause.—An analysis of the 162 cases was made with respect to the four *Ri* groups and to the 2,000-ft. layers above and below the height of the tropopause as illustrated in table 9. Considering all of the HLT cases there does not appear to be any well defined level of maximum of occurrences. Rather there appears to be a fairly constant number of cases ranging from 13 to 17 for each 2,000-ft. layer from 10,000 ft. below to 4,000 ft. above the height of the tropopause. Above this 14,000-ft. layer there is a marked drop-off of cases from 15 to 4 while below this layer the drop-off is from 14 to 8.

All of the cases fall within a 44,000-ft. layer extending 34,000 ft. below to 10,000 ft. above the height of the tropopause. To reduce this layer to a smaller thickness and yet include as large a number of the HLT cases as possible within the layer, the 14,000-ft. layer extending 10,000 ft. below to 4,000 ft. above the tropopause was arbitrarily chosen. Of the 162 HLT cases 105, or about 65 percent fell within this layer. Of these 105 HLT cases, 60 or 57 percent were in the two lower *Ri* groups, and 45 or 43 percent were in the two higher *Ri* groups. Above this layer there were only four cases and all of these were in the two higher *Ri* groups. Below this layer there were 53 cases: 33 or 62 percent in the two lower *Ri* groups and 20 or 38 percent in the two higher *Ri* groups. It appears that there is not too much difference in the correlation between HLT and low *Ri* within the 14,000-ft. layer or below the layer.

TABLE 10.—*Means for the vertical wind shear values (kt./1,000 ft.), lapse rate (° C./1,000 ft.), and* Ri *with respect to the tropopause. For the computation of the mean* Ri *values, 320° C. was used as the mean potential temperature of the layer*

	Means of—			Number of cases	Percent of all cases
	$\left\|\frac{\Delta u}{\Delta z}\right\|$	$\frac{\Delta\theta}{\Delta z}$	Ri		
Below tropopause	4.6	1.5	2.5	126	77.8
Above tropopause	5.3	5.7	7.1	36	22.2

Mean values of the vertical wind shear and stability are shown in table 10. From these the mean Ri values were computed. The mean values for the vertical wind shear were not too different above and below the tropopause with a shear of 5.3 kt. per 1,000 ft. above and a slightly smaller value of 4.6 kt. per 1,000 ft. below. As might be expected the mean values of stability above and below the tropopause are considerably different. Below the tropopause in the troposphere the mean stability was 1.5° C. per 1,000 ft., while above the tropopause in the stratosphere the mean stability was 5.7° C. per 1,000 ft. Even though there was a slightly higher mean vertical wind shear above the tropopause, this was more than offset by the larger mean lapse rate above the tropopause and resulted in a much higher Ri, 7.1, above than below the tropopause, 2.5.

Below the tropopause and to 5,000 ft. below the level of maximum wind.—Thirty cases of the total 162 cases occurred within 5,000 ft. below and up to the tropopause, and 5,000 ft. below or above the level of maximum wind.

The number and percentage of the total cases with $Ri \leq 10$, $Ri \leq 5$, and $Ri \leq 1$ are shown in table 11. It appears that the correlation between low Ri and HLT occurrence is fairly good. Of the 30 possible cases, 25 or 83 percent had $Ri \leq 10$, 21 or 70 percent had $Ri \leq 5$, and 7 or 23 percent had $Ri \leq 1$.

TABLE 11.—*Tabulation of cases with respect to* Ri *which occurred within 5,000 ft. below and up to the tropopause, and within 5,000 ft. below or above the level of maximum wind*

	Number of cases	Percent
$Ri \leq 10$	25	83
$Ri \leq 5$	21	70
$Ri \leq 1$	7	23

TABLE 12.—*Mean values of vertical wind shear (kt./1,000 ft.), lapse rate (° C./1,000 ft.), and* Ri *of cases occurring within 5,000 ft. below and up to the tropopause, and within 5,000 ft. below or above the level of maximum wind*

	All cases
Mean $\|\Delta u/\Delta z\|$	5.2
Mean $\Delta\theta/\Delta z$	1.9
Mean Ri	2.5
Number of cases	30

TABLE 13.—*Percentage of cases with $\Delta\theta/\Delta z \leq 2.4$° C./1,000 ft., $\|\Delta u/\Delta z\| \geq 4.0$ kt./1,000 ft., and* Ri *≤ 5.0 by seasons* at time of occurrence of HLT*

	Fall	Winter	Spring
$\Delta\theta/\Delta z \leq 2.4$° C./1,000 ft.	83	55	79
$\|\Delta u/\Delta z\| \geq 4.0$ kt./1,000 ft.	45	61	45
$Ri \leq 5.0$	55	65	59
Number of cases	40	72	29

*Because of the limited number of summer cases of HLT, 5, no tabulation was made for this season.

The mean values of the vertical wind shear, lapse rate, and Ri are shown in table 12. The relatively low mean Ri of 2.5 is the result of a relatively high vertical wind shear of 5.2 kt. per 1,000 ft., and a relatively low mean stability of 1.9° C. per 1,000 ft.

SEASONAL VARIATION

A detailed breakdown (not shown) by percentage of cases of Ri, lapse rate, and vertical wind shear values was made for 12 hr. before, at, and 12 hr. after the time of occurrence of HLT. For each of these three times, a tabulation of the total cases was made by the four seasons of the year. There was a total of 146 cases; 40 in the fall, 72 in the winter, 29 in the spring, and 5 in the summer. Because of the limited number of summer cases, no discussion will be made for this season.

From the detailed breakdown of Ri, lapse rate, and vertical wind shear values table 13 was tabulated. This table gives the percentage of $Ri \leq 5.0$, lapse rate ≤ 2.4° C. per 1,000 ft., and vertical wind shear ≥ 4.0 kt. per 1,000 ft. for the total cases and for fall, winter, and spring seasons. This tabulation is for time of occurrence.

The results in table 13 indicate that the best correlation between HLT and low Ri was in the winter season, with 65 percent of the HLT cases having $Ri \leq 5.0$. As might be expected, the vertical wind shear parameter also had the best correlation with HLT in the winter, with 61 percent of the HLT cases having a vertical wind shear equal to or greater than 4.0 kt. per 1,000 ft. Unfortunately stability and HLT did not show the best correlation during the winter season. In fact, for the winter season the correlation was the poorest with only 55 percent of the HLT cases having a stability ≤ 2.4° C. per 1,000 ft. The best correlation, 83 percent with the stability parameter occurred in the fall season, but the Ri correlation, 55 percent, was the poorest in this season.

Mean values of Ri, vertical wind shear, and lapse rate for the 146 cases were computed by seasons 12 hr. before, at, and 12 hr. after the time of occurrence and are shown in table 14. The lowest mean values of the Ri and the largest mean values of the vertical wind shear occurred at the time of occurrence for fall, winter, and spring. The mean values of stability showed little change from one 12-hr. period to the next. Only during the fall season did the minimum value of stability occur at the time of occurrence. The winter and spring season had the largest value of stability at the time of occurrence.

Table 14.—*Mean values of vertical wind shear (kt./1,000 ft.), lapse rate (° C./1,000 ft.), and* Ri *with respect to season* 12 hr. before, at, and after the time of occurrence. For the computation of the mean* Ri *values, 320° C. was used as the mean potential temperature of the layer*

Season*	12 hr. before	At	12 hr. after	Number of cases
Mean values of $\|\Delta u/\Delta z\|$				
Fall	2.9	4.0	3.2	40
Winter	3.6	5.4	3.9	72
Spring	2.9	4.5	3.5	29
Mean values of $\Delta\theta/\Delta z$				
Fall	2.1	1.9	2.0	
Winter	2.4	2.9	2.2	
Spring	1.7	2.1	1.7	
Mean values of *Ri*				
Fall	8.8	4.2	6.9	
Winter	6.5	3.5	5.1	
Spring	7.1	3.6	4.9	

*Due to the limited number of summer cases of HLT, 5, no tabulation was made for this season.

3. CONCLUSIONS

In conclusion, the best correlation of high level turbulence occurrence and low *Ri* occurred in the eastern half of the country, below both the level of maximum wind and the tropopause, and in the winter season.

In a majority of the cases, the vertical wind shear parameter appeared to be more important than the lapse rate parameter, which is in agreement with the findings of Mook [8].

In a recent paper Reiter and Hayman [11] questioned the value of the Richardson number as a criterion for clear air turbulence. However, the present study shows, as did Clodman et al. [4], that the *Ri* is a useful parameter under certain meteorological and geographical conditions.

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